ISI PEA & PEB 2021 Compiled by EduSure

1. A number is chosen randomly from the first billion natural numbers. The probability that the product of the number with its two immediate successors is divisible by 24 is closest to

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$

22
23
24

2. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{2n^2+2}{3n^2+1}$, then $\sum_{n=1}^{\infty} a_n$ is

(A) 0

(B) divergent

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

$\lim_{n\to\infty} \frac{2n^2+2}{3n^2+1}$

$\lim_{n\to\infty} \frac{2+\frac{2}{n^2}}{3+\frac{1}{n^2}} = \frac{2}{3}$

3. Each of the four entries of a 2×2 matrix is filled by independently choosing either 1 or -1 uniformly at random. What is the probablity that the matrix is singular?

(A) $\frac{1}{16}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

4. We need to fill a 3×3 matrix by either 0 or 1 such that each row has exactly one 0 and each column has exactly one 0. The number of ways we can do this is

(A) 8 (B) 6

(C) 4 (D) 2

5. Let $f : [0,1] \to \mathbb{R}$ be a convex function with $f(0) = 0$. Which of the following is always true for f?

(A) f is differentiable

(B) f may not be differentiable but it is continuous

(C) $f(x) \geq xf'(x)$ for all $x \in [0,1]$ if f is differentiable

(D) none of the above

$f(x) = x^2$ $\quad xf'(x)$

$f'(x) = x$

2

0 1

6. If $f : \mathbb{R} \to \mathbb{R}$ is strictly quasi-concave, then it follows that

(A) f is not strictly convex

(B) f is not linear

(C) f is monotonic

(D) if f is quadratic, then the coefficient of $x^2 \leq 0$

$y = 1 - x$

7. Consider a function $f : [-1, 1] \to \mathbb{R}$ shown in Figure 1.

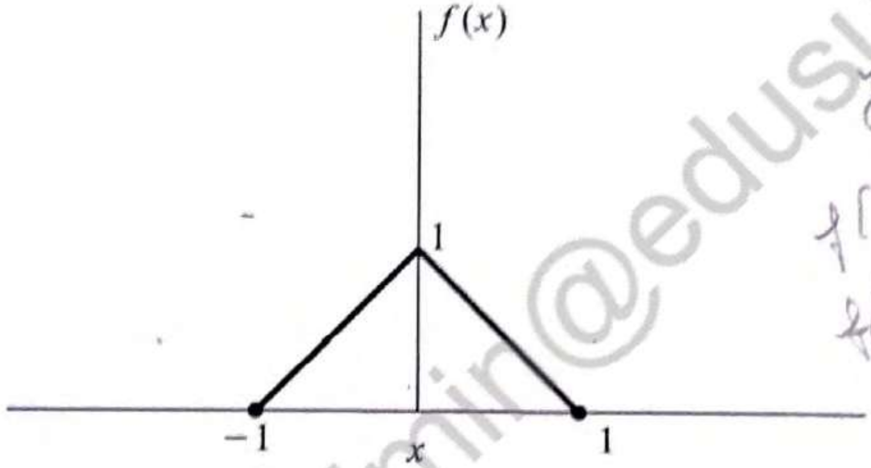


Figure 1: $f : [-1, 1] \to \mathbb{R}$

$y = |x| + 1$

$f(x) = |x| + 1$

$f(x^2 - 1) = |x^2 - 1| + 1 = x^2$

$\frac{1}{2} \times 1 \times 1 \times 2 = 1$

The value of $\int_{-1}^{1} f(x^2 - 1)dx$ equals

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

$f(x^2) =$

8. The rank of the matrix

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

$= 2(3) - 1(0) + 0 = 6 \neq 0$

is

(A) 0 (B) 1

(C) 2 (D) 3

$\int x^2 dx$

9. Let V be the vector space of polynomials $p(x)$ of degree less than or equal to 2 that have real coefficients. Then T is a linear transformation from V to V if T is defined by

$ax^2 + bx + c$

$ax + b > 0$

(A) $T(p(x)) = x + p(x)$

(B) $T(p(x)) = xp(x)$

(C) $T(p(x)) = \frac{dp(x)}{dx}$

(D) $T(p(x)) = \int p(x)dx$ where the constant of integration is taken to be zero.

10. X is a random variable that can take values only in $[0, 10]$. $P(X > 5) \leq \frac{2}{5}$ and $P(X < 1) \leq \frac{1}{2}$. Then

$E(X) = \int x P(x)$

(A) $E(X) \geq 1$

(B) $E(X) \leq 5$

$\int_5^{10} x \frac{2}{5} dx = \frac{x^2}{2} \times \frac{2}{5}$

(C) $E(X) \geq 0.5$ and $E(X) \leq 8.5$

(D) None of the above is true

$\frac{1}{5}[100$

$= 25$

11. Using data from a sample of size n, the intercept and slope coefficients from an ordinary least squares regression of y on x, are a and b respectively. Which of the following is **false**?

$-1 + 0 + 1 = 0$

(A) $\sum_{i=1}^{n}(y_i - a - bx_i)x_i = 0$

(B) $\frac{1}{n}\sum_{i=1}^{n} y_i = a + \frac{b}{n}\sum_{i=1}^{n} x_i$

(C) a and b are the solution to $\min_{\alpha,\beta} \sum_{i=1}^{n}(y_i - \alpha - \beta x_i)^2 = 0$

(D) a and b are the solution to $\min_{\alpha,\beta} \sum_{i=1}^{n} |y_i - \alpha - \beta x_i| = 0$

12. Given data $(-1, 1), (0, 0), (1, 1)$ on (x, y), the standard deviations of x and y and the correlation coefficient of x and y are respectively

(A) $\sigma_x = \sqrt{2}/\sqrt{3}, \qquad \sigma_y = \sqrt{2}/3, \qquad r = 0$

$y = \beta_1 + \beta_2 x_i$

(B) $\sigma_x = 2/3, \qquad \sigma_y = 2/9, \qquad r = 0$

$\hat{y}_i = \beta_1 + \beta_2 \hat{x}_i$

(C) $\sigma_x = 0, \qquad \sigma_y = 0, \qquad r$ is undefined

(D) None of the above

$\sum \hat{u}_i^2 = \sum_{i=1}^{n}(\hat{y}_i - \beta_1 - \beta_2 \hat{x}_i)$

13. An island nation has two potential vaccine firms: denoted as 1 and 2. Both need to invest in R&D to manufacture vaccines. The cost of R&D for firms 1 and 2 are f_1 and f_2 respectively. Once R&D is done, the cost of per unit manufacturing of vaccine is drawn unformly from $[0, 1]$. The firms know their (fixed) cost of R&D but only know that the cost of per unit manufacturing is uniformly drawn from $[0, 1]$..

Total demand of vaccine is 1 unit and if firm $i \in \{1, 2\}$ supplies $q_i \in [0, 1]$ units and has a per unit cost of c_i, it incurs a manufacuring cost of $c_i q_i$ (along with f_i).

Suppose both firms invest in R&D but only the lowest per unit cost firm is chosen to supply the entire one unit of vaccine. What is the total expected cost of vaccination (expected cost is the fixed cost of R&D and expected cost of manufacturing)?

(A) $f_1 + f_2 + \frac{1}{2}$ (B) $f_1 + f_2 + \frac{1}{3}$
(C) $f_1 + f_2 + \frac{2}{3}$ (D) $f_1 + f_2 + \frac{3}{4}$

14. In question 13, suppose $f_1 < f_2 < \frac{1}{6}$. Consider two more alternatives:

(b) Suppose both firms invest in R&D and both supply $\frac{1}{2}$ units of vaccine.

(c) Only firm 1 invests in R&D and supplies the entire one unit of vaccine.

Denote the total expected cost of vaccination from the alternatives (b) and (c) as C_2 and C_3 respectively. Denote the total expected cost of vaccination the alternative in Question 13 as C_1. Then, which of the following is true?

(A) $C_1 < C_2 < C_3$ (B) $C_1 < C_3 < C_2$
(C) $C_2 < C_1 < C_3$ (D) $C_3 < C_1 < C_2$

$c_1 > c_2$

f_1

C_3

$\frac{2}{3}$

15. Consider a production function $z = 2x + 3y$. For what price ratio $\frac{p_x}{p_y}$, will a corner solution in y, i.e. $(x = 0)$ be possible, if the objective is to minimize the cost of producing a given positive quantity z_0 of z?

(A) $\frac{p_x}{p_y} = 2/3$
(B) $\frac{p_x}{p_y} \geq 2/3$
(C) $\frac{p_x}{p_y} < 2/3$
(D) $\frac{p_x}{p_y} \leq -2/3$

16. India and China produce only shirts and phones using only 2 factors of production: either higher skilled labour H or low skilled labour L. Shirts are high skill labour intensive while phones are low skill labour intensive. The production function for each good is identical in both countries. India and China have equal amounts of lower skilled labour, but India has a greater amount of higher skilled labour. Which good will India import?

India China

Shirts — H
Phones — L

(A) Shirts
(B) Phones
(C) Both Shirts and Phones
(D) Neither Shirts nor Phones

17. Continue with the same setup as in Question 16, but now China's population doubles. The overall welfare of the representative agent in India will:

1 2

(A) Increase
(B) Decrease
(C) Stay the same
(D) Increase or decrease

Cost → f_1 f_2

c_1 c_2

$f_1 + f_2 +$

$[0, 1]$

$\frac{1}{2}$ $\frac{1}{2}$

18. Virat and Mithali eat rice and drink milk in exactly the same quantities. The price of rice falls. In response, Virat increases the amount of milk but decreases the amount of rice he consumes. Mithali, on the other hand, increases both rice and milk consumption. Both Virat and Mithali spend all their income on eating rice or drinking milk. For Virat's behaviour to be consistent with standard, well-behaved indifference curves, his preferences over rice consumption imply that for him, rice must be a:

(A) Inferior good
(B) Giffen good
(C) Luxury good
(D) Normal good

19. Continue with the same setup as in Question 18, and choose the answer from below that will correctly fill in the blanks in the following sentence. With respect to rice, for Virat, the income effect must be ___ than the substitution effect while for Mithali the income effect must be ___ than the substitution effect if both Virat and Mithali have standard well-behaved preference relations.

(A) greater, lesser
(B) lesser, lesser
(C) greater, greater
(D) lesser, greater

20. Rohit spends all his money on dosas and filter coffee. He stays in Delhi where each dosa and filter coffee cost the same. He eats 15 dosas and drinks 35 filter coffees in a week. He gets a chance to move to either Chennai or Bangalore. In Chennai, he can just afford to have 40 dosas and 10 filter coffees in a week. Like in Delhi, each dosa and filter coffee cost the same. In Bangalore, he can just afford to have 10 dosas and 20 filter coffees in a week. Here, 2 filter coffees costs the same as 1 dosa. Where will Rohit prefer to stay?

Delhi
15:35

D C
$P_D = P_C$
15 35

(A) Delhi
(B) Chennai
(C) Bangalore
(D) Indifferent between Delhi and Chennai

21. Consider a duopoly with market demand $p = 10 - q$. The cost function of firm 1 is $7q_1$, and that of firm 2 is $2q_2$, where q_i is the quantity produced by firm i, $i = 1, 2$. In equilibrium, firm 2 charges a price of:

(A) 7 (B) 6

(C) 10 (D) 0

22. A cake of size 1 is to be divided among two individuals 1 and 2. Let x_i be the share of the cake going to individual i, $i = 1, 2$, where $0 \leq x_i \leq 1$. The utility functions are $u_1(x_1, x_2) = x_1$, and $u_2(x_1, x_2) = x_2 + |x_1 - x_2|$, where $|a|$ is the absolute value of a. The Pareto optimal cake divisions include:

(A) $(1, 0)$ (B) $(1/2, 1/2)$

(C) $(3/4, 1/4)$ (D) None of the above

23. Inventory investment can be expected to

(A) rise when the real interest rate rises, other things being equal

(B) not depend on the real interest rate, other things being equal

(C) fall when the real interest rate rises, other things being equal

(D) depend only on the change in real GDP

24. Consider the IS-LM model with the real interest rate, R, on the vertical axis and output, Y, on the horizontal axis. Now suppose that the central bank chooses R for the economy, based on its own assessment, at $R = \bar{R}$. In this case the LM curve will

(A) not exist

(B) will be horizontal at $R = \bar{R}$

(C) upward sloping like the usual LM curve

(D) None of the other options

$P = 10 - q$

$R = 10q -$

$MR = 10 -$

$MC = 2$

$10 - 2q =$

$P_D = 2P_C$ 1 : 7 / 1 : 2

25. If the short-run IS-LM equilibrium occurs at a level of income above the natural rate of output, in the long run output will return to the natural rate via

(A) an increase in the price level

(B) a decrease in the interest rate

(C) an increase in the money supply

(D) a downward shift of the consumption function

26. If the short-run aggregate supply curve is steep, the Phillips curve will be:

(A) flat

(B) steep

(C) backward-bending

(D) unrelated to the slope of the short-run aggregate supply curve

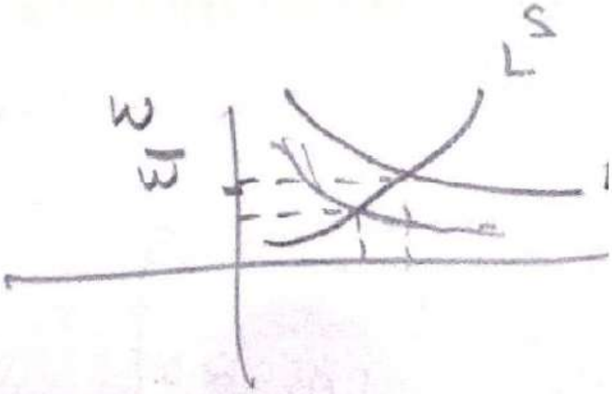
27. Consider a supply-demand diagram for the labor market with an upward sloping labor supply curve (L^s) and a downward sloping labor demand curve (L^d). Let the wage be on the vertical axis, and the level of employment (L) be on the horizontal axis. Suppose the wage is rigid above the equilibrium wage at $\bar{w}$, i.e., it fails to adjust to clear the labor market. Then a reduction in labor demand leads to

(A) A larger reduction in employment compared to the case if wages were flexible

(B) A smaller reduction in employment compared to the case if wages were flexible

(C) The same reduction in employment compared to the case if wages were flexible

(D) None of the other options.



28. Let the economy's production function be given by $Y = AK^{\frac{1}{3}}L^{\frac{2}{3}}$ where Y = output, $A > 0$ is the level of technology (also called total factor productivity), K is the capital stock, and L is the level of employment. Consider the standard supply-demand diagram for labor as in the previous question, except that there are no wage rigidities this time. The labor demand curve is given by ____ ; a negative total factor productivity shock leads to a ____in labor demand, and a ____in employment.

$$\frac{\partial Y}{\partial L} = \frac{2}{3} AK^{1/3} L^{-1/3}$$

(A) $w = \frac{3}{2}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; upward shift; fall

(B) $w = \frac{3}{2}\frac{AK^{\frac{2}{3}}}{L^{\frac{1}{3}}}$; downward shift; fall

(C) $w = \frac{2}{3}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; downward shift; fall

(D) $w = \frac{1}{3}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; downward shift; rise

29. Suppose there are two countries, B and C, that have no trade and no financial transactions with any countries except each other. B imports a total of goods worth 10 million bollars from C, where a bollar is a unit of B's currency. B has no exports. Which of the following must be true?

(A) B has a capital account deficit ✗

(B) C has a current account deficit ✗

(C) C is buying assets from B.

(D) The exchange rate of collars per bollar is bigger than 1, where a collar is a unit of C's currency.

$$\frac{C}{B} > 1$$

$C > B$

30. There are no capital controls between the US and the UK. If the interest rate is higher in the US than in the UK, then we can conclude that

(A) The US dollar is expected to appreciate with respect to the pound (the UK's currency)

(B) The pound is expected to appreciate with respect to the US dollar

(C) The interest rate in the US is expected to increase

(D) The interest rate in the US is expected to decrease

Group A

1. Let $f : \Re \to \Re$ be a continuously differentiable function which has at least three distinct zeros. (We say x is a zero of f if $f(x) = 0$). Let $g : \Re \to \Re$ be defined as follows: $g(x) = e^{x/2} f(x)$ for all $x \in \Re$.

   (i) Prove that g has at least three distinct zeros.

   (ii) Prove that the function $f + 2f'$ has at least two distinct zeros.

(10+20=30)

2. (i) Consider the following two variable optimization problem:

$$\max_{x,y} \; (x^2 + y^2)$$

subject to

$$x + y \leq 1$$

$$x, y \geq 0$$

Find all solutions of this optimization problem.

   (ii) In a kingdom far, far away, a King is in the habit of inviting 1000 senators to his annual party. As a tradition, each senator brings the King a bottle of wine. One year, the Queen discovers that one of the senators is trying to assassinate the King by giving him a bottle of poisoned wine. Unfortunately, they do not know which senator, nor which bottle of wine is poisoned, and the poison is completely indiscernible. However, the King has 10 prisoners he plans to execute. He decides to use them as taste testers to determine which bottle of wine contains the poison. The poison when taken has no effect on the prisoner until exactly 24 hours later when the infected prisoner suddenly dies. The King needs to determine which bottle of wine is poisoned by tomorrow so that the festivities can continue as planned. Hence he only has time for one round of testing. How can the King administer the wine to the prisoners to ensure that 24 hours from now he is guaranteed to have found the poisoned wine bottle?

(12+18=30)

3. (i) Let A and B be matrices for which the product AB is defined. Show that if the columns of B are linearly dependent, then the columns of AB are linearly dependent.

(ii) Let e_i denote the column vector with three elements, each of which is zero, except for the i-th element, which is 1. Consider a linear transformation $L : \Re^3 \to \Re^3$ with $L(e_1) = e_1$, $L(e_2) = e_1 + e_2$, and $L(e_3) = e_2 + e_3$. Does L map $\Re^3$ onto $\Re^3$? Prove your answer.

(15+15=30)

Group B

4. This question pertains to a situation in which a particular commodity, like rice, is both available at a subsidised rate from a fair price shop (ration shop) and at a higher price from the open market. Suppose a consumer can buy a certain (fixed) quantity of rice at a lower price from the ration shop (that is, there is a ration quota). In addition, he can buy more of rice (assume a uniform quality of rice) from the open market at a higher price. (You may assume that consumers preferences are represented by standard downward sloping, smooth, convex indifference curves.)

   (i) Graphically depict the consumer's equilibrium (assuming he exhausts the ration quota and in addition buys from the open market).

   (ii) Suppose rice is a normal good. What will happen to the quantity of rice purchased from the open market (over and above the ration quota) in equilibrium if there is a cut in the ration quota? Briefly explain.

   (iii) Suppose rice is a normal good. What will happen to the quantity purchased in the open market (over and above the ration quota) if the subsidised price (price at which the ration quota rice could be bought) is increased (but is still lower than the open market price)? Will your conclusion change if rice is an inferior good? Briefly explain.

(10+10+10=30)

5. There are plenty of fish in the Dull Lake. Boats can be hired by fishermen to catch fish and sell it on the fish market. The revenue earned each month from a total of x boats is given by the following expression: Rupees $10,000\{4x - \frac{1}{2}x^2\}$. Each boat costs Rupees 20,000 each month.

   (i) Derive the marginal and average revenue per boat

   (ii) The Dull municipality is considering giving out permits for each boat that fishes so they can track who is fishing from the lake. If these permits are allocated freely, how many boats will fish every month?

   (iii) If total profit is to be the maximum possible, how many boats should fish every month?

   (iv) Dull municipality decides to charge for the permits instead of giving them out for free. What should the per-boat charge for the permit be if total profits are to be the maximum possible?

(5+10+5+10=30)

6. The Shoddy Theater screens movies every week and is located on a university campus which has only students and faculty as residents. It is the only source of watching movies for both faculty and students, and is large enough to accomodate all faculty and students. Faculty demand for movie tickets is given by $500 - 4P_F = Q_F$, where P_F refers to the price of the ticket paid by faculty and Q_F refers to the number of tickets purchased by faculty. Demand by students is described by $100 - 2P_S = Q_S$, where P_S refers to the price of the ticket paid by students and Q_S refers to the number of tickets purchased by students. The cost to service demand equals 500.

   (i) If the price charged is to be the same for faculty and students, what price would Shoddy Theater set in order to maximize its profits?

   (ii) Now imagine that Shoddy Theater decides to charge different prices for faculty and students. What would these prices be, if Shoddy Theater wants to maximize profits?

(15+15=30)

Group C

7. Consider a Solow type economy, producing a single good, according to the production function:

$$Y(t) = K(t)^{\alpha} \; L(t)^{(1-\alpha)}$$

Where, $0 < \alpha < 1$, and $Y(t), K(t)$ and $L(t)$ are the output of the good, input of capital, and input of labour used in the production of the good, respectively, at time t. Capital and labour are all fully employed.

Labour force grows at the exogenous rate, $\eta > 0$, i.e.,

$$\frac{1}{L(t)} \frac{dL(t)}{dt} = \eta > 0.$$

Part of the output is consumed and part saved. Let $0 < s < 1$ be the fraction of output that is saved and invested to build up the capital stock. Also assume that there is no depreciation of capital stock.

Then it follows that:

$$sY(t) = \frac{dK(t)}{dt}$$

Where $\frac{d}{dt}$ is the time derivative.

With this above given description of the economy, one can find out the steady state growth rate of Y, for this economy. Growth rate of output is given by: $\frac{1}{Y(t)} \frac{dY(t)}{dt} \equiv g_Y$.

Assume, the economy begins at date 0, from a per capita capital stock, $k(0) \equiv \frac{K(0)}{L(0)} < k^*$, where k^* denotes the steady state per capita capital stock.

(i) Demonstrate formally, whether the growth rate of output (g_Y), at the beginning date 0, is greater, equal or less than the steady state growth rate of output.

(ii) For the same economy, consider, two alternative beginning date scenarios: with per capita capital stock, given by: Case 1. $k(0)$; Case 2. $k'(0)$. Where, $k(0) < k'(0) < k^*$ Can you compare the beginning date growth rates of output in the two cases?

(iii) Next, consider two Solow type economies, namely, A and B. They are isolated from each other and are working on their own. Both the economies have absolutely the same description as given before, except for the fact that the fraction of income saved in country A, denoted by s_A is greater than the fraction of income saved in country B, denoted by s_B. Let $k^A(0)$ be the initial date per capita capital stock in country A and $k^B(0)$, the initial date per capita capital stock in country B, which are both less than their respective steady state values. Assume, $K^A(0) < K^B(0)$. Can you figure out whether the initial date growth rate of output in country A is greater than, equal or less than the initial date growth rate of output in country B? In case you find the data provided to you is insufficient to make any comment on this, please point it out.

(20+5+5=30)

8. (i) What is the money multiplier? What determines its size? What is the relation between the monetary base, the money multiplier, and the money supply? Which of these variables can the central bank change to change the money supply? What is the direction of change in each case?

(ii) Why might the cash/deposit ratio and the reserve to asset ratio be decreasing functions of the rate of interest? How does an interest-sensitive money supply affect the LM curve? Illustrate with a diagram, comparing this LM curve with the standard LM. How does this change the effectiveness of counter-cyclical fiscal policy (in a closed economy)? Explain.

(15+15=30)

9. (i) What is the difference between the real and the nominal exchange rate? Give an example to explain this to someone who has not studied economics. Is an increase in the real cost of imports an improvement or a deterioration in the terms of trade?

(ii) A small open economy has a government budget surplus and a trade deficit. Explain whether there is a private sector surplus, deficit or balance. Examine the consequences in the short run for output, the trade balance and the government budget balance of a sudden fall in private consumption in this economy (due to an epidemic in the small country) under (a) fixed exchange rates, (b) flexible exchange rates. Use the Mundell-Fleming model with perfect capital mobility. Explain the adjustment mechanisms.

(10+20=30)